• Discuss Problems 1 & 3 from Midterm 2

• Hand out Questionnaires about Midterm

• New topic "Rigid Motions" (Section 11 of your Textbook)

**Def:** A rigid motion is an act of taking an object and moving it from the starting position to some ending position without altering its shape/size.

**Note:** The distance between any 2 points is preserved under applying a rigid motion.

**Def:** Given an initial object (starting position of it), we say that two rigid motions are equivalent if they "move" the object to the same ending position.

**Notations:**
- We will use curly letters, e.g. \( M, N, A, B \)… to denote rigid motion.
- Given a point \( P \) of an object and a rigid motion \( M \), we will denote the ending position of \( P \) by \( P' \) and call it the image of \( P \) under the rigid motion \( M \).

**Def:** Given a rigid motion \( M \) and a point \( P \), we say that \( P \) is a fixed point of the rigid motion \( M \) if \( P' = P \).

*For 2-dimensional objects moving in a plane, there are only 4 types of rigid motions and we will discuss this next.*
1st type: "Reflections"

A reflection in the plane is a rigid motion that moves an object into a new position that is a mirror image of the starting position.

The role of a mirror is played by a line called the axis of reflection. In other words, given any line \( l \) (the axis of reflection), we consider reflection of any point. To do that, consider a line \( l' \) perpendicular to \( l \) and passing through \( P \). Then \( P' \) is also on that line, on the opposite side of \( l \), so that the distances \( PA = PA' \)

\( (A = \text{intersection point of } l \& l') \)

**Example:**

![Reflection Example Diagram]

**Key Observation**

You can recover an axis of reflection \( l \) given any point \( P \) and its image \( P' \) such that \( P' \neq P \). For this you just need to construct a line passing through the middle-point of \( PP' \) and perpendicular to \( PP' \).
Key Properties of Reflections

1. Reflection is completely determined by its axis \( l \).
2. Reflection is completely determined by a point \( P \) and its image \( P' \) (assuming \( P' \neq P \)).
3. Fixed points of a reflection are exactly the points of its axis.
4. Applying the same reflection twice, we always get the same position of an object as we started from.
   In other words, Reflection \(^2 = \) Identity map
5. Reflections are improper rigid motions
   changes orientation

- Discuss several examples in class