Lecture 28

2nd type of rigid motions in 2d: "Rotations"

A rotation in the plane is a rigid motion that pivots/swings an object around a fixed point $O$. It is defined by 2 pieces of information:
1. point $O$ - the rotocenter,
2. angle of rotation.

Example:

Key properties of rotations

- The rotocenter is the only fixed point of any rotation, which is not an identify map.
- The rotation is completely determined by $O$ and $\alpha$.
- The rotation is completely determined by any two points $P, Q$ and their images $P', Q'$ (as long as $P \neq P', Q \neq Q'$).

The rotocenter $O$ is obtained as intersection of two lines $l$ and $l'$ which pass through middle points of $PP'$, $QQ'$ and are perpendicular to them.

The angle of rotation $\alpha = \angle POP'$.

A $360^\circ$ rotation is the identity motion $\Rightarrow$ rotation by $\alpha = n \cdot 360^\circ + \beta$ ($n$-integer) is equivalent to a rotation by $\beta$ with the same rotocenter.

A rotation is a proper rigid motion, i.e. preserves orientation.
3rd type of rigid motion: "Translations"

A translation consists of essentially dragging an object in a specified direction and by a specified amount. These two pieces of information are combined in the form of a vector of translation.

A vector is represented by an arrow pointing in the direction of translation, while its length is the length of translation.

Example

Key properties of translations:

- A translation is completely determined by any point P and its image P'.

\[ \overrightarrow{P} = \overrightarrow{P'} \]

- A translation (which is not an identity map) has no fixed points.

- A translation is a proper rigid motion, i.e., it preserves an orientation.

- A translation by \( \overrightarrow{v} \) followed by a translation in the opposite direction by \( -\overrightarrow{v} \) is the identity map.

Ex: Find an image of the shape F under a translation given P and P':

Discuss several more examples!