Today: "Gnomons"

Two objects are said to be similar if one is a scaled version of the other.

Examples:

1. Any two squares are similar

2. Any two disks are similar

3. Two triangles with sides \( a, b, c \) and \( a', b', c' \) are similar if and only if there exists a scaling factor \( k > 0 \) such that \( a' = ka, b' = kb, c' = kc \)

Remark: They are also similar if and only if they have the same angles.

4. Two rectangles with sides \( a, b \) and \( a', b' \) are similar if and only if there exists a scaling factor \( k > 0 \) such that \( a' = ka, b' = kb \) or equivalently \( \frac{a}{b} = \frac{a'}{b'} \)

Remark: We assume \( a \leq b \), \( a' \leq b' \) or otherwise we could also have \( a' = kb, b' = ka \).

Key definition: A gnomon \( G \) to a figure \( A \) is a connected figure which when suitably attached to \( A \) produces a new figure similar to \( A \). We use \( A \& G \) to denote a figure obtained by attaching \( G \) to \( A \).

Example 1: If \( A = \) the disk of radius \( r \), then \( G = \) is obviously a gnomon to \( A \), since \( A \& G = \) the disk of radius \( R \).
Q: Which rectangles have square gnomons?

Consider a rectangle $R = a \begin{array}{c} \hline b \end{array}$ and assume that the square $G = b \begin{array}{c} \hline b \end{array}$ is a gnomon of $R$, i.e. $a \begin{array}{c} \hline b \end{array}$ is similar to $b \begin{array}{c} \hline b \end{array}$.

Then $\frac{b}{a} = \frac{b+a}{b} \Rightarrow$ the sides of $R$ are in a divine proportion (see Lecture Notes #30). In other words, $\frac{b}{a} = \phi$ - the golden ratio. $\Rightarrow R$ is a golden rectangle.

P.S.: Given two figures $A$ and $B$ such that $B$ is a scaled version of $A$ with the scaling factor $k > 0$, we have:

(a) Perimeter ($B$) = $k \cdot$ Perimeter ($A$)

(b) Area ($B$) = $k^2 \cdot$ Area ($A$)

Discuss these 2 properties!!!

Above we defined the Golden rectangles.

Analogously, one can define Fibonacci rectangles as rectangles whose sides are consecutive Fibonacci numbers.

Note: Starting from a Fibonacci rectangle with sides $F_{n-1}$ and $F_n$, and attaching to it a square of size $F_n \times F_n$, we get a Fibonacci rectangle with sides $F_n$ and $F_{n+1}$:

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\begin{array}{c} \hline \ \rc F_n \cr \ \rc \ \ \ F_{n+1} \end{array} \quad (F_{n-1} + F_n = F_{n+1})