Lecture #35

- Last time:
  - \( nP_r = \frac{n!}{(n-r)!} \) number of permutations on \( r \) objects from a set of \( n \) objects.
  - \( \binom{n}{r} \) number of combinations of \( r \) objects chosen from a set of \( n \) objects.

**Ex.1** (Exercise 16.2.22): Nine people (4 men and 5 women) line up at a checkout stand in a grocery store. (a) In how many ways can they line up? (b) In how many ways can they line up if the first person in line must be a woman? (c) In how many ways can they line up if they must alternate (woman, man, woman, man,...)?

- (a) \( 9! \)  
- (b) \( 5 \cdot 8! \)  
- (c) \( 5! \cdot 4! \)

**Ex.2** (Exercise 16.2.32): Bob has 20 different dress shirts in his wardrobe.

(a) In how many ways can Bob select seven shirts to pack for his business trip?  
(b) In how many ways can Bob select 5 of those 7 dress shirts he packed: one for Monday, one for Tue,..., one for Friday?

- (a) \( \binom{20}{7} = \frac{20!}{13! \cdot 7!} = 77,520 \)  
- (b) \( \frac{7!}{2!} = 2520 \)
last time: Probability assignment

* Ask the audience if anyone remembers the definition.

Recall: Probability assignment is a function that assigns to each event $E$ a number between 0 and 1, which represents the probability of the event $E$, denoted $P_c(E)$.

**Properties:**
1. $P_c(\emptyset) = 0$
2. $P_c(S) = 1$
3. $P_c(E \cup E') = P_c(E) + P_c(E')$ if $E \cap E'$ is empty.

Due to the property (3), if $S$ is finite, i.e. $S = \{x_1, \ldots, x_n\}$, then $P_c$ is uniquely determined by specifying $0 \leq P_c(x_1), \ldots, P_c(x_n) \leq 1$ such that $P_c(x_1) + \ldots + P_c(x_n) = 1$.

Then the value $P_c(E)$ is obtained summing up the probabilities of the individual outcomes that make up the event.

**Def.:** The combination of the sample space $S$ and the probability assignment is called a probability space.

**Ex. 3 (Exercise 16.3.33):** Consider the sample space

$S = \{x_1, x_2, x_3, x_4, x_5\}$. Suppose $P_c(x_1) = 0.22$, $P_c(x_2) = 0.24$.

(a) Find the probability assignment for the probability space when $x_4, x_5, x_6$ all have the same probability.

(b) Find the probability assignment for the probability space when $P_c(x_5) = 0.1$ and $x_3$ has some probability as $x_4, x_5$.
Def: A probability space in which each simple event has an equal probability is called an equiprobable space. If \( N \) is the size of the sample space, then we see that each individual outcome has probability \( \frac{1}{N} \). More generally, if \( k = \text{size of } E \), \( N = \text{size of } S \), then in an equiprobable space \( P(E) = \frac{k}{N} \).

Ex 4 (Exercise 16.3.33) An honest coin is tossed three times in a row. Find the probability of the following events.

(a) \( E_1: \) "the coin comes up heads exactly twice"
(b) \( E_2: \) "all three tosses come up the same"
(c) \( E_3: \) "exactly half of the tosses come up heads"
(d) \( E_4: \) "the first two tosses come up tails".

\[
\begin{align*}
(a) &\quad \frac{3}{8}, \\
(b) &\quad \frac{2}{8} = \frac{1}{4}, \\
(c) &\quad 0, \\
(d) &\quad \frac{2}{8} = \frac{1}{4}
\end{align*}
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It is also very instructive to consider Example 16.21 from your textbook.